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Neutrino masses in composite little Higgs model

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Abstract: The composite little Higgs model, a UV completion for the SU(5)/SO(5) little Higgs model, incorporates supersymmetry into strong gauge dynamics. We extend the study of flavor physics in the model, and find that it is similar to the bosonic technicolor model. Lepton flavor violations and neutrino mass matrix arise once R-parity violating superpotential is introduced to the model, as in the MSSM. We identify various low-energy effective $\Delta L = 2$ lepton flavor violating operators, and find that most of them are similar to those of the R-parity violating MSSM. There is a new operator which involves only leptons and the pseudo-Nambu Goldstone bosons of the little Higgs model. We further study a possibility that this operator gives a dominant contribution to the neutrino mass matrix.

Keywords: Neutrino Physics, Beyond Standard Model, Higgs Physics, Technicolor and Composite Models.

Contents

1. Introduction

In the Standard Model (SM) there are four particles whose masses are still unknown, the Higgs boson, h and the three neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$. The direct searches at LEP set the lower bound on the SM Higgs mass, $\sim 114\,\mathrm{GeV}$ [1] while precision electroweak data, obtained at the colliders (LEP, SLD, C0, CDF) in the framework of the SM, suggest the SM Higgs boson must be relatively light and its upper limit is $\sim 210\,\mathrm{GeV}$. This gives us a hope that the SM Higgs is light enough to be discovered at the LHC in near future. On the other hand, through the past few years, several neutrino experiments have identified convincingly that neutrinos are indeed massive and like the quarks, mix among themselves [2]. This is a solid evidence of the new physics beyond the SM.

There are several popular mechanisms for understanding small neutrino masses. In a generic see-saw mechanism, very massive right-handed neutrinos are integrated out at high energy and neutrino mass is inversely proportional to the mass scale of the right-handed neutrino, M_R and is quadratic in the Higgs *vev*, $m_\nu \sim v^2/M_R$ where v is the *vev* of the SM Higgs. On the contrary, triplet scalar models admit a neutrino mass which is linear to the *vev* of the Higgs triplet, $m_\nu \sim yv'$ where y is a dimensionless coupling constant [3]. As a third way, in supersymmetric theories a tiny neutrino mass [4] is induced by R-parity violating (RPV) superpotential.

In all these mechansims, the relevant Higgs are assumed to be fundamental particles with masses. But there is a different possibility that the Higgs is a pseudo-Nambu-Goldstone boson (pNGB). Three years ago, Arkani-Hamed, Cohen, Katz, and Nelson constructed the successful Higgs model with pNGBs, called the "the littlest Higgs model" [5, 6]. In the littlest Higgs model, the SM Higgs doublet belongs to a set of pNGBs in a global SU(5)/ SO(5) nonlinear sigmal model, and other elements of the pNGBs are grouped into a complex Higgs triplet. The nonlinear transformation of the pNGBs under the collective global symmetries naturally ensures the absence of the SM Higgs mass term of the form $m^2|h|^2$. However, at the electroweak scale the SM Higgs potential is induced by the Coleman-Weinberg potential [7] in the gauge sector as well as in the fermion sector, and the SM Higgs mass is generated by loop contributions from the massive particles.

In the present article, we focus on neutrino physics in the SU(5)/ SO(5) little Higgs model. To account for small neutrino masses, the pNGBs interact with neutrinos. But it is a drawback that the UV cutoff of the little Higgs models is relatively low, being typically in the range of $10\,\mathrm{TeV} \lesssim \Lambda \lesssim 100$ TeV. Thus we suggest that there is an unknown symmetry that almost forbids the couplings between the pNGBs and neutrinos in a UV completion of the little Higgd models. We search for the mechanism which introduces small neutrino masses. So far there have been two known UV completions of the little Higgs models: one is the composite little Higgs model (CLHM) [8], and the other is suggested by the ADS/CFT correspondence [9]. Both models are based on the gauge group $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times U(1)_Y$, and need strong dyamics at the TeV scale. Here we further investigate the CLHM to account for small neutrino masses.

The outline of the rest of the paper is as follows: In section 2 we review the Higgs sector of the SU(5)/ SO(5) little Higgs model, and take into account the low-energy effective $\Delta L = 2$ lepton flavor violating (LFV) operator. In section 3 we review the composite little Higgs model, and further investigate flavor physics in the composite little Higgs model. In section 4 we introduce R-parity violation (RPV) in the composite little Higgs model, and then identity various $\Delta L = 2$ LFV operators. In section 5, we investigate neutrino mass matrix from the LFV operators coming from PRV couplings. In section 6, we specify the case where the LFV operator, coupled only to the pNGB, gives a dominant neutrio mass matrix. Finally, we draw a conclusion in section 7.

2. SU(5)/ SO(5) little Higgs model

The littlest Higgs model begins with a global SU(5) symmetry, with a locally gauged subgroup $[\mathrm{SU}(2)_1 \times \mathrm{U}(1)_1] \times [\mathrm{SU}(2)_2 \times \mathrm{U}(1)_2]$ [5]. The SU(5) global symmetry breaking is spontaneously broken down to its subgroup SO(5) at the scale $f \sim 1\,\mathrm{TeV}$ resulting in fourteen NGBs. This breaking arises from a *vev* of the 5×5 symmetrical matrix Ξ, which transforms $\Xi \to V \Xi V^T$ under SU(5).

$$\Sigma_0 = \begin{pmatrix} & & \mathbf{1}_{2\times 2} \\ & 1 & \\ \mathbf{1}_{2\times 2} & & \end{pmatrix}. \tag{2.1}$$

The four NGBs are eaten by the gauge bosons so that the gauge group $[\mathrm{SU}(2)\times \mathrm{U}(1)]^2$ is broken down to the electroweak gauge group $\mathrm{SU}(2)_L\times \mathrm{U}(1)_Y$. The remaining ten NGBs can be parameterized by the non-linear Σ field

$$\Sigma = e^{i\Pi/f}\Sigma_0 e^{i\Pi^T/f} = e^{2i\Pi/f}\Sigma_0\,, \qquad \Pi = \begin{pmatrix} & h^\dagger/\sqrt{2} & \phi^\dagger \\ h/\sqrt{2} & & h^*/\sqrt{2} \\ \phi & h^T/\sqrt{2} & \end{pmatrix}, \tag{2.2}$$

and consist of $h = (2, 1/2)$ and $\phi = (3, 1)$ in $\mathrm{SU}(2)_L\times \mathrm{U}(1)_Y$:

$$h = \begin{pmatrix} h^+ & h^0 \end{pmatrix}, \qquad \phi = \begin{pmatrix} \phi^{++} & \phi^+/\sqrt{2} \\ \phi^+/\sqrt{2} & \phi^0 \end{pmatrix}. \tag{2.3}$$

The h is identified as the SM Higgs while the ϕ is an addition to the SM. The mechanism of collective symmetry breaking generates the potential for ϕ and h: ϕ acquires a large mass of order gf from one-loop gauge quadratic divergent part of the Coleman-Weinberg potential, while h acquires a negative mass squared parameter from logarithmically enhanced parts of the Coleman-Weinberg potential in the gauge sector, in the scalar sector and in the fermion (third generation up-type quark) sectors. The low energy effective potential admits *vev*'s for h and ϕ: $\langle h^0\rangle = v/\sqrt{2}$, and $\langle \phi^0\rangle = -iv'$.[1]

The littlest Higgs model has an ambiguity of two U(1)'s charge assignements in the fermion sector [6]. To avoid this ambiguity and an extra heavy U(1) gauge field at low energy, one can consider a simpler model, in which only one U(1) is gauged. This makes it easier to build a UV completion of a SU(5)/ SO(5) little Higgs model. In the following, we consider the SU(5)/ SO(5) little Higgs model with the $\mathrm{SU}(2)_1\times \mathrm{SU}(2)_2\times \mathrm{U}(1)_Y$ gauge group. The ungauged field, corresponding to the extra U(1) gauge field in the littlest Higgs model, remains an exact NGB to the order in which we are working. The impact of this axion-like field at the electroweak scale is disccussed in ref. [10]. We neglect this axion-like field in the following.

We now take into account the interactions between Σ and the SM leptons. The SM lepton doublet and singlet are transformed as $\ell = (2, 1)$ and $e^c = (1, 1)$ in the $\mathrm{SU}(2)_1\times \mathrm{SU}(2)_2$ gauge group. To account for mass of the charged leptons, the LH model contains the Yukawa interaction of the form [5]

$$\mathcal{L}_{Yuk} = \frac{1}{2}\lambda^e_{\alpha\beta} f \epsilon^{ijk}\epsilon^{xy}(\ell_i)^\alpha \Sigma^*_{jx}\Sigma^*_{ky}(e^c)^\beta + \text{h.c.} \tag{2.4}$$

where α, β represents the generation and i, j, k, x, y represent the component in the SU(5) representation. Note that i, j are summed over 1,2 (index in the gauge group $\mathrm{SU}(2)_1$), $k = 3$, and x, y are summed over 4, 5. As we expand (2.4) in power of $1/f$, we read the SM Yukawa couplings at leading order.

There may be other interactions between Σ and the SM leptons which admit lepton flavor violations to account for tiny neutrino masses observed in neutrino oscillation experiments. Provided that neutrino mass is induced primarily from Σ field, one can consider a

[1] From contraint on ρ parameter, $v' \ll v$.

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LFV operator of the form

$$\mathcal{L}_{LFV} = z_{\alpha\beta}\epsilon^{ij}\epsilon^{kl} f(\bar{\ell_i^c})^\alpha \Sigma^*_{jk}(\ell_l)^\beta + \text{h.c.} \tag{2.5}$$

where $z_{\alpha\beta}$ are couplings, and i, j, k, l are summed over 1,2 (component in the SU(2)$_1$ representation). Note that (2.5) violates lepton flavor number by two units, and preserves the SU(3)$_2$ global symmetry in the lower 3×3 block of Σ, while breaking the SU(3)$_1$ global symmetry in the upper 3×3 block of Σ. We expand eq.(2.5) to leading order in powers of $1/f$:

$$\begin{aligned}\mathcal{L}_{LFV} = -z_{\alpha\beta}\Bigg[&\bar{\nu^c}_\alpha\nu_\beta\left(2i\phi^0 + \frac{1}{f}h^0h^0\right) - (\bar{\nu^c}_\alpha e_\beta + \bar{e^c}_\alpha\nu_\beta)\left(\sqrt{2}i\phi^+ + \frac{1}{f}h^+h^0\right) + \\ &+ \bar{e^c}_\alpha e_\beta\left(2i\phi^{++} + \frac{1}{f}h^+h^+\right)\Bigg] + \text{h.c.}\end{aligned} \tag{2.6}$$

Note that the operators are associated with the triplet appear at dimension 4 while operators associated with the doublet appear at dimension 5.

After h and ϕ develop *vev*'s, mixing occurs between them (See the details in Appendix.). The low energy effective operator is given in terms of the mass eigenstates of the higgs and the longitudinal components of the gauge fields

$$\begin{aligned}-z_{\alpha\beta}\Bigg\{&2\bar{\nu^c}_\alpha\nu_\beta\left(v' + \tfrac{v^2}{4f}\right) - \tfrac{\bar{\nu^c}_\alpha e_\beta + \bar{e^c}_\alpha\nu_\beta}{\sqrt{2}}\left[2\left(1 - \tfrac{v'}{f} - \tfrac{2v'^2}{v^2}\right)\Phi^+ + \tfrac{4}{f}\left(v' + \tfrac{v^2}{4f}\right)G^+\right] + \\ &+ \bar{e^c}_\alpha e_\beta\left[2\Phi^{++} + \tfrac{1}{f}\left[\left(1 - 4\tfrac{v'^2}{v^2}\right)G^+G^+ - 4\tfrac{v'}{v}G^+\Phi^+ + 4\tfrac{v'^2}{v^2}\Phi^+\Phi^+\right]\right]\Bigg\} + \text{h.c.}\end{aligned} \tag{2.7}$$

where Φ^+ and Φ^{++} are the singly charged and doubly charged scalars, and G^+ is the Goldstone boson that is eaten by the W^+ boson, giving it a mass. Note that the $\bar{\nu^c}\nu$ associated term gives Majorana masses to neutrinos, so the neutrino mass matrix is then given by

$$[m_\nu]_{\alpha\beta} = 2z_{\alpha\beta}\left(v' + \frac{v^2}{4f}\right). \tag{2.8}$$

Lightness of neutrinos arises from $z_{\alpha\beta}$. As shown in eq. (2.8), the h's *vev* acts like a ϕ's *vev* at order v^2/f. There is a relation between the two *vev*'s by demanding the Higgs triplet mass squared to be positive, $v' \lesssim v^2/4f$. Furthermore, the current experimental limits on the ρ parameter lead to more stringent constraint on the Higgs triplet *vev*, $v' \lesssim \frac{1}{10}\frac{v^2}{f}$ [11]. As a consequence, the Higgs doublet contribution to neutrino mass is larger than the Higgs triplet contribution: $[m_\nu]_{\alpha\beta} \lesssim z_{\alpha\beta}v^2/f$.

The absolute scale of neutrino masses is not determined by the neutrino oscillations, but can be determined by the observation of the end-point part of the electron spectrum of Tritium β-decay, the observation of large-scale structures in the early universe, and the dectection of the neutrinoless double beta decay. From these observations, one can set the upper limit on the heaviest neutrino mass at the $0.1 \sim 1$ eV scale. Then the upper bound of the coupling constants is estimated by

$$|z_{\alpha\beta}| \lesssim 10^{-12}\left[\frac{f}{1\,\text{TeV}}\right]. \tag{2.9}$$

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	SO(7)	$SU(3)_c$	$SU(2)_2$	$SU(2)_1$	$U(1)_Y$
L	1	1	1	2	-1/2
E^c	1	1	1	1	1
Q	1	3	1	2	1/6
U^c	1	$\bar{3}$	1	1	-2/3
D^c	1	$\bar{3}$	1	1	1/3
$\Phi_{\bar{3}}$	7	$\bar{3}$	1	1	-2/3
Φ_3	7	3	1	1	2/3
$\Phi_{2'}$	7	1	2	1	-1/2
Φ_2	7	1	1	2	1/2
Φ_0	7	1	1	1	0
Y	1	$\bar{3}$	1	2	-7/6
Y'	1	3	2	1	7/6
H_u	1	1	1	2	1/2
H_d	1	1	1	2	-1/2

Table 1: Quantum numbers of chiral superfields.

It is such a tiny number that one may raise a question on its origin. In the next section, we will study the composite little Higgs model to find a successful mechanism for the tiny coupling.

3. Composite little Higgs model

The idea of composite Higgs was first introduced by Dugan, Georgi, and Kaplan [12]. The composite little Higgs model was first introduced as a UV completion of the SU(5)/ SO(5) little Higgs model with the $SU(2)_1 \times SU(2)_2 \times U(1)_Y$ gauge group. The NGBs arising from the SU(5)/ SO(5) global symmetry breaking are fermion condensation through a strong SO(7) gauge group, called Ultra-color. The strong gauge dynamics are merged into supersymmetry so that the number of supermultiplets in the CLHM is larger than that in the MSSM. The relevant energy scales in the composite little Higgs model are shown in figure 1. Note that the UV cutoff scale of the SU(5)/ SO(5) little Higgs model is the same as the supersymmetry breaking scale. The quantum numbers of matter superfields in the theory are listed in table 1.

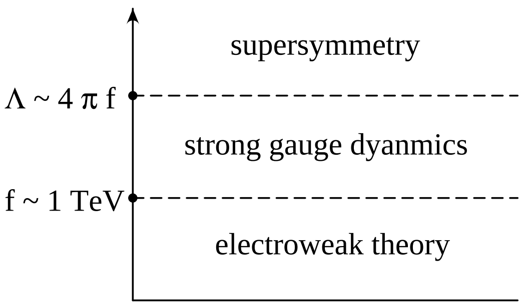

Figure 1: Energy scale in the composite little Higgs model.

Let us review the SU(5)/ SO(5) symmetry breaking in detail. The (approximate) SU(5) global symmetry of the little Higgs model acts on the fields $\Phi_{2,2',0}$. When the Ultra-color forces become strong at the scale $\Lambda \sim 4\pi f$, bilinears of Ultra fermions $\tilde{\phi}_{0,2,2'}$ condense [5] as

$$\langle \tilde{\phi}_\rho \tilde{\phi}_\sigma \rangle \approx 4\pi f^3 S_{\rho\sigma} \,. \tag{3.1}$$

By comparing hypercharge of bilinears of the condensates with that of the Σ field in the littlest Higgs model, one can see the correspondence of the Ultrafermion condensates in the composite little Higgs model to the Higgs fields in the littlest Higgs model. See the table 2.

compostie little Higgs	littlest Higgs
S_{22}	ϕ
$S_{2'2'}$	$\phi^\dagger$
S_{02}	h
$S_{02'}$	$h^\dagger$

Table 2: Correspondence of Ultrafermion condensates in the composite little Higgs model to the Higgs fields in the littlest Higgs model.

All the SM particles are neutral under Ultra-color. The chiral superfields $\Phi_{0,2,2',3,\bar{3}}$ are charged under Ultra-color, as technifermions in the technicolor model. H_u and H_d are up-type and down-type Higgs superfields, respectively, as in the MSSM. The scalar components of the Higgs fields are irrelevant to the SM Higgs. Y and Y' are introduced to evade the gauge anomalies in the theory. Their interactions with other supermultiplets are either suppressed by their large masses or forbidden by some unknown global symmetry. For example, a renormalizable superpotential of the form QYE^c is highly supressed by proton decay.

We now describe the origin of the Yukawa interactions for the *light* quarks[2] and leptons. For leptons, the Yukawa couplings emerge from (2.4), and the charged leptons acquire masses from the *vev* of the composite Higgs doublets $S_{02}, S_{02'}$. For down type quarks, the Yukawa couplings are given in the same fashion as in the leptons. In contrast, the Yukawa interactions for up-type quarks are obtained in slightly different ways: (i) Σ is replaced by its complex conjugate, Σ^*, and (ii) the third generation mixes with vector-like fermions with a TeV mass scale, and the top quark becomes much heavier than other SM quarks.[3] In order to introduce light quark and lepton masses in a renormalizable theory, we couple the quarks and leptons to the Higgs superfields H_u and H_d in the same fashion as in the MSSM.

We now describe Yukawa couplings for the light quarks and leptons. The Yukawa couplings are governed by the the superpotential

$$W = \mu H_u H_d + \lambda^u_{\alpha\beta} H_u Q_\alpha U^c_\beta + \lambda^d_{\alpha\beta} H_d Q_\alpha D^c_\beta + \lambda^e_{\alpha\beta} H_d L_\alpha E^c_\beta + \omega H_d \Phi_2 \Phi_0 \,, \tag{3.2}$$

and soft supersymmetry breaking scalar terms

$$V_{soft} = -(\mu B H_u H_d + h.c.) + M_u^2 |H_u|^2 + M_d^2 |H_d|^2 + M_0^2 |\Phi_0|^2 + M_2^2 |\Phi_2|^2 + \cdots \,, \tag{3.3}$$

[2] A quark that is lighter than the top quark.

[3] Heavy composite spin 1/2 fermions are fermion condensate of the form $\tilde{\phi}_\rho \tilde{\phi}_\sigma \lambda$, where either ρ or σ is 3 or $\bar{3}$, and λ is a gaugino of Ultra-color. Among them charge 2/3 vector-like fermion mixes with the fundamental quark in the third generation.

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where we take $M_{u,d}, M_{0,2}$ to be real and positive, and omit soft-supersymmetry-breaking mass terms of the SM sfermions and gluino which are nearly irrelevant to flavor problem. The B parameter is important in that it couples between H_u and H_d bosons. We adjust the over-all phase of H_u and H_d so that B is real and positive. For simplicity, we assume that all the mass parameters in (3.2) are of the scale Λ:

$$\{|\mu|, B, M_{u,d}, M_{0,2}\} \sim \Lambda\,. \tag{3.4}$$

The $\lambda^{u,d,e}$ parameters are the Yukawa couplings of the (light) up quark, down quark and lepton superfields at high energy, respectively. Note that the ω term is an addition to the general renormalizable (R-parity conserving) superpotential, and its main role is to link the SM fermions and the Ultrafermions at low energy via H_d. For simplicity, we take ω be real and positive by adjusting the over-all phase of Φ_0 and Φ_2.

Now let us relate the Yukawa couplings above the scale Λ to those at the electroweak scale. To do so, we briefly describe the physics from the top down. At the scale Λ, massive superpartners of the SM fields and Ultrascalars decouple. At the same scale, the Ultra-color forces become strong so that the Ultrafermion condensate is triggered, and the SU(5) $\rightarrow$ SO(5) symmetry breaking occurs. That is, the composite Higgs triplet and composite Higgs doublet are formed as a set of NGBs. At the scale f, the composite Higgs triplet gets a mass of order gf. Below the scale f, one gets couplings of the light quarks and leptons to the composite Higgs doublet to the leading order as follows:

$$y^u_{\alpha\beta} q_\alpha f S_{02} u^c_\beta + y^d_{\alpha\beta} q_\alpha f S^*_{02} d^c_\beta + y^e_{\alpha\beta} \ell_\alpha f S^*_{02} e^c_\beta + \text{h.c.} \tag{3.5}$$

where $y^{u,d,e}$ are the Yukawa couplings at low energy. At the electroweak scale, the composite Higgs doublet acquires a *vev* so that EWSB takes place, and the light quarks and leptons acquire masses as proceeding via Ultrascalar exchange, as shown in figure 2.[4] Thus one estimate the Yukawa couplings for the light quarks and leptons at low energy as follows:

$$y^e_{\alpha\beta} \approx \frac{\omega\lambda^e_{\alpha\beta}}{4\pi}\frac{\Lambda^2}{M_d^2+|\mu|^2} \sim \frac{\omega\lambda^e_{\alpha\beta}}{8\pi}\,, \tag{3.6}$$

$$y^d_{\alpha\beta} \approx \frac{\omega\lambda^d_{\alpha\beta}}{4\pi}\frac{\Lambda^2}{M_d^2+|\mu|^2} \sim \frac{\omega\lambda^d_{\alpha\beta}}{8\pi}\,, \tag{3.7}$$

$$y^u_{\alpha\beta} \approx \frac{\omega\lambda^u_{\alpha\beta}}{4\pi}\frac{\mu B\Lambda^2}{(M_u^2+|\mu|^2)(M_d^2+|\mu|^2)} \sim \frac{\omega\lambda^u_{\alpha\beta}}{16\pi}\,, \tag{3.8}$$

where we have used (3.4). In order that Yukawa couplings for the up type (light) quark are comparable in size to those for the down type quark, one confirm the previous assumption, $B \sim M_{u,d}$. Note that the low-energy Yukawa couplings are approximately, by a factor of $\sim \omega/8\pi$ or $\omega/16\pi$, proportional to the high-energy Yukawa couplings.

Additional dangerous operators which could violate flavor at low energy must be proportional to the *only* sources of flavor violation, namely the matrices $\lambda^{u,d,e}$. For example,

[4] One may note that this scenario is very similar to the bosonic technicolor model (BTM) [13], which merges supersymmetry and technicolor into a theory to resolve the phenomenological difficulties that supersymmetry and technicolor separately have.

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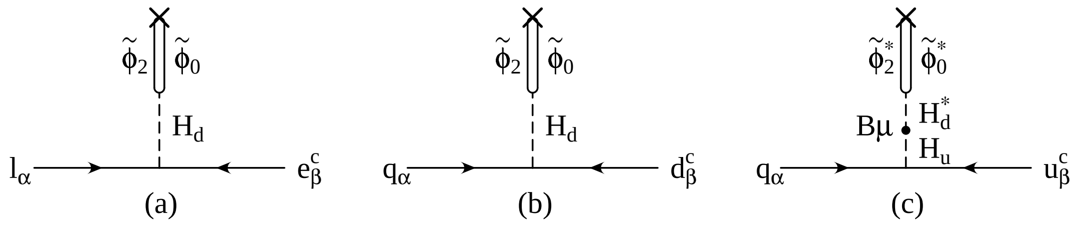

Figure 2: Diagrams contributing to the Yukawa couplings of (a) charged leptons, (b) down type quarks, and (c) up type quarks.

the next leading flavor violating diagrams come from gauge one-loop corrections which are suppressed by a factor of $\frac{g^2}{16\pi^2}$ compared with the leading diagrams. One finds that additional dangerous operators which could violate flavors are much suppressed, so the low energy Yukawa matrices are nearly aligned with the matrices $\lambda^{u,d,e}$.

Given the low-energy Yukawa couplings alone one cannot determine the value of ω because the high-energy Yukawa couplings are still unknown. This provides more uncertainties to the parameter spaces in the composite little Higgs model at high energy. However, ω is constrained by the largest mass among the light quarks. The b quark must be taken for the purpose and its Yukawa coupling is $y^d_{33} \lesssim \sqrt{2}m_b/v$. From (3.7), ω should be constrained as follows:

$$\omega|\lambda^d_{33}| \approx 0.6\,. \tag{3.9}$$

As for the SUSY flavor problem, we take the masses of all the SM superpartners and Ultrascalars be the UV cutoff scale, $\Lambda \sim 10\,\mathrm{TeV}$ or above so that little or no squark and slepton mass degeneracy is required at low energy to satisfy the phenomenological constraints on CP conserving flavor changing neutral currents (FCNC) [14]. Furthermore, the *vev*'s of $H_{u,d}$ in the composite little Higgs model are much smaller than that in the MSSM,[5] which gives an extra suppression in the FCNC processes. Constraints associated with one-loop contributions to the neutron electric dipole mement, involving gaugino and squark exchange, are also satisfied even if CP violating phases are of order one. Thus, in this scenario, one can naturally resolve the SUSY flavor problem.

However, there are potentially dangerous sources of FCNC due to a heavy charge 2/3 quark, which is required to give the top quark a large mass. The CLHM explains the origin of the vector-like quark and its companions whose masses, if any, lie in the TeV range. The presence of the heavy vector-like quark induces non-vanishing mixing angles in the neutral currents sector, and they may give significant contributions to the FCNC processes at tree level compared to that from the SM, but all the predictions are beyond the experimental sensitivity in near future [15].

4. R-parity violations in the composite little Miggs model

As in the R-parity violating MSSM [4], the CHLM does not distinguish between the down-type Higgs superfield H_d and the lepton superfields L_α with respect to charges under the

[5]We will show it in the next section.

gauge group. As a result, it is convenient to denote these four supermultiplets by one symbol $L_m \equiv (H_d, L_e, L_\mu, L_\tau)$. In the followings we use Greek indices for the usual three dimensional lepton flavor space and Latin indices m, n for the four dimensional extended lepton flavor space.

Now one generalize (3.2) to the superpotential with R-parity violations:

$$W = \mu_m L_m H_u + \lambda^u_{\alpha\beta} H_u Q_\alpha U^c_\beta + \lambda^d_{m,\alpha\beta} L_m Q_\alpha D^c_\beta + \lambda^e_{mn,\alpha} L_m L_n E^c_\alpha + \omega_m L_m \Phi_0 \Phi_2 \quad (4.1)$$

where μ and ω in (3.2) are now extended to four-component vectors $\mu_m = (\mu, \mu_\alpha)$ and $\omega_m \equiv (\omega, \omega_\alpha)$ in the extended lepton flavor space, respectively. $\lambda^d_{m,\alpha\beta}$ is a vector in a similar fashion while $\lambda^e_{mn,\alpha}$ becomes a antisymmetric tensor under the intercahnge of the indices m, n. By adjusting the phases of the lepton superfields L_α one can make either μ_α or ω_α positive and real. In what follows we take ω_α be real and positive. Assuming that the effects of R-parity violation is quite small, one expect that $|\mu_\alpha| \ll |\mu|$ and $\omega_\alpha \ll \omega$. Note that the small RPV parameters depend on the basis choice for these superfields although physical observables are independent of the choice of basis. In addition, we include soft supersymmetry breaking potential

$$V_{soft} = -(\mu B_m L_m H_u + h.c.) + M_u^2 |H_u|^2 + (M^2_{\tilde{L}})_{mn} \tilde{L}^{i*}_m \tilde{L}^i_n + (M^2_{\tilde{E}})_{\alpha\beta} \tilde{E}^*_\alpha \tilde{E}_\beta + \cdots , \quad (4.2)$$

where the B parameter in the CLHM is now extended to a four component, $B_m \equiv (B, B_\alpha)$, with $|B_\alpha| \ll B$ due to small R-parity violation. $B, M_u, M_{\tilde{L}}$, $[(M_{\tilde{L}})_{00} \equiv M_d], M_{\tilde{E}}$ are all of order Λ. Further, we omit other RPV soft-supersymmetry-breaking potentials which appear in the MSSM.

As mentioned before, at the scale f the composite Higgs doublet drives small *vev*'s of $H_{u,d}$ so that the W and Z acquire small mass contributions from the *vev*'s of $H_{u,d}$. In contrast, above the scale Λ the *vev* of composite Higgss doublet has not turned on yet, so the *vev*'s of $H_{u,d}$ is expected to be zero. We drive the argument by considering the neutral scalar potential for sneutrinos, H_u and H_d. The contribution of the neutral scalar fields to the scalar potential is given by

$$\begin{aligned} V_{neutral} &= \left(M_u^2 + |\hat{\mu}|^2\right) |H_u|^2 + \left[\left(M^2_{\tilde{L}}\right)_{mn} + \mu_m \mu_n^*\right] \tilde{\nu}_m \tilde{\nu}_n^* - \\ &- \left(\mu B_m \tilde{\nu}_m H_u + \mu B_m^* \tilde{\nu}_m^* H_u^*\right) + \frac{1}{8}\left(g_1^2 + g_Y^2\right)\left[|H_u|^2 - |\tilde{\nu}_m|^2\right]^2 \end{aligned} \quad (4.3)$$

where $\tilde{\nu}_m \equiv (H_d, \tilde{\nu}_\alpha)$, $|\hat{\mu}|^2 = \sum_m |\mu_m|^2$, and g_1 and g_Y are the gauge couplings in $SU(2)_1$ and $U(1)_Y$ respectively. The *vev*'s for the neutral scalars denoted by $\langle H_u \rangle \equiv \frac{v_u}{\sqrt{2}}$ and $\langle \tilde{\nu}_m \rangle \equiv \frac{v_m}{\sqrt{2}}$, are determined by the following minimization conditions

$$\left(M_u^2 + |\hat{\mu}|^2\right) v_u^* = \mu B_m v_m - \frac{1}{8}(g_1^2 + g_Y^2)(|v_u|^2 - |v_d|^2) v_u^* \quad (4.4)$$

$$\left(\left(M^2_{\tilde{L}}\right)_{mn} + \mu_m \mu_n^*\right) v_n^* = \mu B_m v_u + \frac{1}{8}\left(g_1^2 + g_Y^2\right)\left(|v_u|^2 - |v_d|^2\right) v_m^* \quad (4.5)$$

with $|v_d|^2 \equiv \sum_m |v_m|^2$. Up to this point, there is no preferred direction in the extended lepton flavor space so that we can choose, for convenience, a basis where all the sneutrino *vev*'s vanish such that $v_m = (v_d, 0, 0, 0)$.[6] In this basis, without changing the over-all phase of $H_{u,d}$ we adjust their relative phases so that v_u is real. Then eqs. (4.4) and (4.5) show that v_d is also real so that these equations become

$$(M_u^2 + |\hat{\mu}|^2) v_u = \mu B v_d - \frac{1}{8}(g_1^2 + g_Y^2)(v_u^2 - v_d^2) v_u \,, \tag{4.6}$$

$$(M_d^2 + |\mu|^2) v_d = \mu B v_u + \frac{1}{8}(g_1^2 + g_Y^2)(v_u^2 - v_d^2) v_d \,, \tag{4.7}$$

$$((M_{\tilde{L}}^2)_{\alpha 0} + \mu_\alpha \mu^*) v_d = \mu B_\alpha v_u \,. \tag{4.8}$$

To satisfiy the assumption (3.4) with the *vev*'s of $H_{u,d}$, $v_{u,d} \ll f$, there is *only* one solution for the Higgs *vev*'s, $v_u = v_d = 0$. That is, the gauge group is unbroken above the scale Λ. However, the $\mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2$ gauge groups are broken into its diagonal subgroup $\mathrm{SU}(2)_L$ at the scale f and the broken gauge bosons acquire mass of order gf. Furthermore, at the electroweak scale the SM gauge groups are broken and the W and Z bosons acquire masses.

The contributions to the W and Z masses arise after the *vev*'s of the Ultrafermion condensate are turned on. The W and Z bosons acquire masses from different two sources; the leading contribution comes directly from the *vev* of the composite Higgs doublet, v and the other indirectly from the *vev* of $H_{u,d}$ which are driven by the composite Higgs. But the latter is much smaller than the former. The ratio of the *vev* of H_d to the *vev* of the composite Higgs doublet is given by

$$\frac{v_d}{v} \approx \frac{\frac{\omega}{4\pi}\frac{\Lambda^2}{M_d^2 + |\mu|^2} v}{v} \sim \frac{\omega}{8\pi} \ll 1 \,, \tag{4.9}$$

where we have assumed $\Lambda \sim M_d$. One can impose severe contraint on the ratio by precision electroweak data. The ω parameter can not be arbitrarily small because it should meet the condition (3.9) as well. By taking $\lambda_{33}^d \sim 1$, we set ω in the range of $0.1 \lesssim \omega \lesssim 1$. For $\omega = 1$, we estimate the upper limits on the $\lambda^{d,e}$ and list them in table 3.

λ	upper limit
d	1×10^{-3}
s	2×10^{-2}
b	0.6
e	7×10^{-5}
μ	1×10^{-2}
τ	1×10^{-2}

Table 3: Upper limits on the elements on the Yukawa matrices at high energy for $\omega = 1$.

5. Contributions to the neutrino masses

In the previous section, we have shown that the RPV terms allow lepton number violation by one unit. Two of these are taken together to construct the low-energy effective operators, which violate lepton number by two units. The neutrino mass matrix arises from both tree- and loop-level diagrams, as in the RPV MSSM. Many RPV parameters are involved in these operators. In the RPV MSSM, there is a region of the RPV parameter spaces where the bilinear μ_α term dominantly contributes to the neutrino mass matrix in order naturally to describe the neutrino mass hierarchy in neutrino oscillation experiments [17]:

[6] We use the basis-independent parameters constructed in and write the neutrino mass matrix in terms of various PRV parameters $\delta_\mu^\alpha, \delta_B^\alpha, \delta_{\lambda^d}^{m\alpha\beta}$ and $\delta_{\lambda^e}^{mn\alpha}$ [16]. In the basis where the sneutrino *vev*'s are zero, the parameters correspond to $\mu_\alpha/|\mu|$, $B_\alpha/|B|$, $\lambda_{m\alpha\beta}^d$ and $\lambda_{mn\alpha}^e$, respectively.

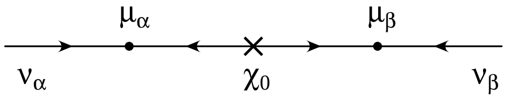

Figure 3: Tree level neutrino mass in the mass insertion approximation. A blob represents mixing between the neutrino and the up-type Higgsinos/gauginos. The cross on the neutralino propagator signifies a Majorana mass term for the neutrino.

the bilinear term gives mainly the largest neutrino mass and the one-loop contributions are subordinate, inducing other light neutrino masses.

The composite little Higgs model has an additional RPV paramter compared with the RPV MSSM. This is the ω_α parameter. Though the ω_α-term is trilinear it behaves as a bilinear term below the scale f. Among all the contributions to the neutrino mass matrix, only the effective opeartor consisting of the ω_α term appears in the SU(5)/ SO(5) little Higgs model. In the absence of the μ_α contribution to the neutrino mass, the ω_α term can, in turn, provide another economical framework for the solution of neutrino mass hierarchy. In the following we quantitatively analyze the tree- level contributions while we qualitatively comment on the loop-level contributions to neutrino masses.

5.1 Tree level ($\mu\mu$) contribution

In the previous section, we have shown that μ extended to a four-vector, μ_m, and this admits bilinear RPV interactions between leptons and Higgsinos,

$$\mu_\alpha L_\alpha H_u + \text{h.c.} \tag{5.1}$$

Two of these interactions are joined together and then $\Delta L = 2$ LFV interactions are induced through mixing with the neutralinos as shown in figure 3.

As in a generic RPV MSSM, the 7×7 gaugino-Higgsino-neutrino mass matrix in a basis spanned by the two neutral gauginos, the higgisinos, and three generations of neutrinos is non-diagonal:

$$M^{(n)} = \begin{pmatrix} M_1 & 0 & \frac{1}{2}g_Y v_u & -\frac{1}{2}g_Y v_d & 0 & 0 & 0 \\ 0 & M_2 & -\frac{1}{2}g_1 v_u & \frac{1}{2}g_1 v_d & 0 & 0 & 0 \\ \frac{1}{2}g_Y v_u & -\frac{1}{2}g_1 v_u & 0 & -\mu & -\mu_1 & -\mu_2 & -\mu_3 \\ -\frac{1}{2}g_Y v_d & \frac{1}{2}g_1 v_d & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{5.2}$$

where $M_{1,2}$ are the gaugino mass parameters, and mass hierarchy among the parameters is $M_{1,2} \sim \mu \gg v \gg v_d$.

Assuming that all the masses of gauginos and higgisinos are of order 10 TeV or higer, one can integrate out all of them at low energy. Thus the mass matrix for remaining neutrinos is estimated by

$$[m_\nu]^{(\mu\mu)}_{\alpha\beta} \approx \left[\frac{(g_1 v_d)^2}{4M_2} + \frac{(g_Y v_d)^2}{4M_1}\right]\left(\frac{\mu_\alpha \mu_\beta}{\mu^2}\right). \tag{5.3}$$

This constitutes a rank 1 mass matrix, leading to only one nonzero mass eigenvalue which is presumably m_3:

$$m_3 \approx \left[\frac{(g_1 v_d)^2}{4M_2} + \frac{(g_Y v_d)^2}{4M_1}\right]\left(\frac{\sum_\alpha |\mu_\alpha|^2}{\mu^2}\right). \tag{5.4}$$

Note that it is suppressed by a huge scale difference between Majorana gaugino mass and the down type Higgs *vev*. In contrast, they are same order in the RPV MSSM. For $m_3 \lesssim 0.1$ eV, $|\mu| \sim M_{1,2} \sim 10\,\mathrm{TeV}$, and $g_1 v_d \sim g_Y v_d \lesssim 10\,\mathrm{GeV}$,[7] the upper bound of the RPV parameter $\delta^\alpha_\mu (\equiv \mu_\alpha/|\mu|)$ is estimated by

$$\delta^\alpha_\mu \lesssim 10^{-4}\left(\frac{10\text{ GeV}}{g_1 v_d}\right)\left(\frac{M_{1,2}}{10\text{ TeV}}\right)^{1/2}. \tag{5.5}$$

Note that upper bound of δ^μ_α is sensitive to the down-type Higgs *vev* : The smaller the down type Higgs *vev* v_d is, the larger the RPV parameter ratio δ^μ_α is. However, v_d is, as described in (4.9), not arbitrarily small. On the contrary, large value of δ^μ_α is not preferred due to the assumption of small RPV parameter in the beginning.

5.2 One-loop neutrino masses

Combination of two among $\Delta L = 1$ LFV interactions in the superpotential and in the soft supersymmetry breaking scalar mass terms gives rise to a one-loop induced neutrino mass with the down squark (slepton) and antisquark(antislepton) pairs being exchanged in the loops along with their ordinary partners, just as in the RPV MSSM [4, 17, 19]. Any one-loop contribution can not induce the LFV operator in the SU(5)/ SO(5) little Higgs model due to no couplings to the Ultrafermions. The squarks (sleptons) in the composite little Higgs model have larger masses of order 10 TeV, and the upper bounds on the RPV parameters, in general, are less than those in the RPV MSSM.

In the following, we summarize the approximate expressions of various one-loop contributions to neutrino mass matrix [18, 19]:

$$[m_\nu]^{(\mu\lambda^d)}_{\alpha\beta} \sim \sum_\gamma \frac{3g}{16\pi^2} m_{d_\gamma} \frac{\mu_\alpha \lambda^d_{\beta,\gamma\gamma} + \mu_\beta \lambda^d_{\alpha,\gamma\gamma}}{\Lambda}, \tag{5.6}$$

$$[m_\nu]^{(\lambda^d\lambda^d)}_{\alpha\beta} \sim \sum_{\gamma,\delta} \frac{3}{8\pi^2} \lambda^d_{\alpha,\gamma\delta}\lambda^d_{\beta,\delta\gamma} \frac{m_{d_\gamma} m_{d_\delta}}{\Lambda}, \tag{5.7}$$

$$[m_\nu]^{(\lambda^e\lambda^e)}_{\alpha\beta} \sim \frac{1}{8\pi^2} \lambda^e_{\alpha,\gamma\delta}\lambda^e_{\beta,\delta\gamma} \frac{m_{l_\gamma} m_{l_\delta}}{\Lambda}, \tag{5.8}$$

$$[m_\nu]^{(BB)}_{\alpha\beta} \sim \frac{g^2}{64\pi^2} \frac{B_\alpha B_\beta}{\Lambda}\epsilon, \tag{5.9}$$

where we ignore the $(\mu\lambda^e)$ and $(\lambda^e\lambda^e)$ loop contributions due to Yukawa suppression of the down type quarks and a suppression factor, ϵ' in (μB) loops are expected to be $\lesssim 1$. The (μB) and (BB) loops are least constrained so that they may give the largest loop contributions. Taking, $\delta^\alpha_\mu \sim 10^{-4}$, this leads to $\delta^\alpha_B (\equiv B_\alpha/B) \sim 10^{-7}$. It is interesting to

[7]Here we set $\omega \approx 1$ and $g_1 \lesssim 1$.

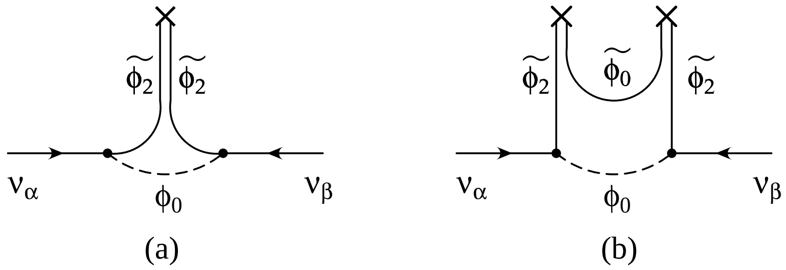

Figure 4: Diagrams generating the LFV operator, (2.5). (a) represents interaction with the composite Higgs triplet. (b) represents interacton with the composite Higgs doublet.

note that the required sizes of the δ's are too small, where as we expect them to be naturally of order one. The smallness of δ's can be understood in the framework of some horizontal symmetries which are spontaneously broken by *vev*'s of some "flavoron" fields, as in the RPV MSSM [18]. In order to maintain neutrino mass hierarchy in a simple framework, we still take the assumption that one-loop diagrams contribute insignificantly to the heaviest neutrino mass in the presence of large tree-level contributions.

5.3 $(\omega\omega)$ contribution

Now we focus on the ω_α-term in (4.1), which is a trilinear interaction:

$$\omega_\alpha \epsilon_{ij} L^i_\alpha \Phi^j_2 \Phi_0 + \text{h.c.} \tag{5.10}$$

where we write down explicitly dependence of the gauge indices $i, j = 1, 2$. Two of these interactions are combined together, and $\Delta L = 2$ LFV interactions are induced below the scale Λ through the Ultrafermions condensation. The two diagrams shown in figure 4 give rise to neutrino mass matrix below the scale f. Taking the mass of the Ultrascalar ϕ_0 be of the scale $\Lambda \gtrsim 10\,\text{TeV}$ as in ref. [8], one has the neutrino mass matrix as follows,

$$[m_\nu]^{(\omega\omega)}_{\alpha\beta} \approx \frac{\omega_\alpha\omega_\beta}{4\pi}\frac{\Lambda^2}{M_0^2}\left(v' + \frac{v^2}{4f}\right) \approx \frac{\omega_\alpha\omega_\beta}{4\pi}\left(v' + \frac{v^2}{4f}\right). \tag{5.11}$$

This also constitutes a rank 1 mass matrix, leading to *only* one nonzero eigenvalue,

$$m_3 \approx \frac{1}{4\pi}\left(v' + \frac{v^2}{4f}\right)(\omega_1^2 + \omega_2^2 + \omega_3^2)\,. \tag{5.12}$$

Comparing it with the definition of $z_{\alpha\beta}$ in (2.7) one write down the coupling constant in terms of ω_α

$$z_{\alpha\beta} = \frac{\omega_\alpha\omega_\beta}{8\pi}\,. \tag{5.13}$$

6. Leading $(\omega\omega)$ contribution in neutrino masses

In the present section, we consider the region in the RPV parameter spaces where the $(\omega\omega)$ contribution to the neutrino mass dominates over all the other contributions. This is an interesting region because the neutrino mass is almost irrelevant to the UV completions of the SU(5)/ SO(5) little Higgs model. From eq. (5.11), the neutrino mass matrix elements depend both on the ratio of the Ultrascalar mass to the UV cutoff scale and on the three ω_α's. By rescaling the ω_α, $\omega_\alpha \to \frac{M_0}{\Lambda}\omega_\alpha$, one sees the ratio disappear in eq. (5.11). Thus, without knowledge of the Ultrascalar mass and the scale Λ one can analyze the neutrino mixing angles only with ω_α's.

The neutrino ν_α in the weak eigenstates are mixtures of the neutrino ν_i in mass eigenstates with the mass m_i $(i = 1, 2, 3)$

$$\nu_\alpha = \sum_i U_{\alpha i}\nu_i\,, \tag{6.1}$$

where $U_{\alpha i}$ is a 3×3 unitary mixing matrix parameterized in the Kobayashi-Maskawa manner. Then the diagonalization of the neutrino mass matrix is given by

$$U^\dagger[m_\nu]U = \mathrm{diag}[m_1, m_2, m_3]\,, \tag{6.2}$$

where we choose a basis where neutrino mass eigenstates are in ascending order of mass, (m_1, m_2, m_3). With that in mind, we analyze the neutrino mixing angles in terms of ω_α's. One gets a simple relation for $\hat\omega_\alpha(\equiv \omega_\alpha/\sqrt{\omega_1^2+\omega_2^2+\omega_3^2})$ from eqs. (5.11) and (6.2):

$$\hat\omega_\alpha\hat\omega_\beta = U\mathrm{diag}\,(0,0,1)U^T \tag{6.3}$$

where the mixing matrix U is parameterized as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} + s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}. \tag{6.4}$$

Here we ignore CP phase because the three ω_α are, by definition, all real parameters. Then we identify $\hat\omega_\alpha$ with the mixing angles:

$$\hat\omega_1 = s_{13}\,, \qquad \hat\omega_2 = s_{23}c_{13}\,, \qquad \hat\omega_3 = c_{23}c_{13}\,. \tag{6.5}$$

With the experimental data from the recent neutrino oscillation experiments [2],

$$\begin{aligned} \Delta m_{32}^2 &= 2.2\times10^{-3}\ \mathrm{eV}^2\,, & & \Delta m_{21}^2 = 8.1\times10^{-5}\ \mathrm{eV}^2\,,\\ \sin^2\theta_{23} &= 0.50\,, \qquad \sin^2\theta_{12} = 0.30\,, & & \sin^2\theta_{13} < 0.068\,, \end{aligned} \tag{6.6}$$

one obtains the approximate values of $\hat\omega_\alpha$'s:

$$\hat\omega_1 \approx 0\,, \qquad \hat\omega_2 = \hat\omega_3 \approx \frac{1}{\sqrt{2}}\,, \tag{6.7}$$

which implies that[8]

$$\omega_2 \approx \omega_3 \approx 4\times10^{-6}\,, \qquad \omega_1 \ll \omega_2\,. \tag{6.8}$$

This leads to $\delta_\omega^\alpha(\equiv \omega_\alpha/\omega) \sim 10^{-6}$.

There are special regions of the parameter spaces where μ_α or B_α are parallel to ω_α. In these regions, neutrino mass hierarchy is still maintained. For more complicated situations, i.e. the μ_α tree-level diagram or one loop-level diagrams are competitive in amplitude with the ω_α diagram, one can consider degenerate neutrino mass patterns.

[8]we set $f = 1\,\mathrm{TeV}$, $v' \ll \frac{v^2}{4f}$.

7. Conclusion

The SU(5)/ SO(5) little Higgs model describes the SM Higgs as a subset of pNGBs, which implies the UV cutoff is typically in the range of $10 \lesssim \Lambda \lesssim 100\,\mathrm{TeV}$. We have constructed a low-energy effective operator which describes lepton flavor violation in the SU(5)/ SO(5) little Higgs model. After acquiring the *vev*'s, the Higgs doublet and the Higgs triplet give rise to a neutrino mass matrix. The origin of the small neutrino mass is not understood in the model itself.

The composite little Higgs model is a UV completions of the SU(5)/ SO(5) little higgs model, which combines strong gauge dynamics with supersymmetry. We have further worked out flavor physics of the composite little Higgs model, in particular, the fermion mass generation via four-fermi interaction between ordinary fermions and Ultra fermions. The mediators of the four-fermi interaction are the Ultrascalars with mass of order 10 TeV or higer, which naturally explains the suppression of FCNC at low energy.

R-parity violation is incorporated into the composite little Higgs model such that lepton flavor violation and neutrino mass are naturally explained. We have identified various lepton flavor violating operators with R-parity violating interactions, and have studied the region of the RPV parameter spaces by contraints on the upper limits on the heaviest neutrino mass. In particular, we have identified the presence of the low-energy LFV effective operator in the SU(5)/ SO(5) little Higgs model, and have analyzed a special region of RPV parameter spaces where $(\omega\omega)$ contribution to neutrino mass matrix dominantes over other contributions.

One can further ask questions on lepton violating processes like $\mu \to e\gamma$ and $\mu \to 3e$ in the composite little Higgs model. We expect that the prediction of these processes are much lower than the current experimental bounds because the Ultrascalar mass is larger than the squark and slepton masses in the MSSM, and further the relevant couplings z's are very small.

Finally, we would like to comment on the virtue of the composite little Higgs model. Though the MSSM is the most realistic supersymmetric model it suffers from a naturalness problem, called the 'supersymmetric little hierarchy problem'. Little Higgs was introduced as an alternative to supersymmetry. Combining little Higgs with supersymmetry is a possible path to avoid the naturalness problem. One can apply both supersymmetry and strong gauge dynamics to find a UV completions of the other little Higgs models. The first thing to do is to find a correct strong guage dynamics which contains the pNGBs of the model as fermion condensates.

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A. Mixing in scalar sector

The gauge eigenstates of the Higgs fields h^+ and ϕ^+ can be written in terms of the mass eigenstates of the Higgs fields G^+ and Φ^+ as follows:

$$\begin{aligned}
h^0 &= (c_0 H - s_0 \Phi^0 + v)\sqrt{2} + \frac{i\left(c_P G^0 - s_P \Phi^P\right)}{\sqrt{2}}\,, \\
\phi^0 &= (s_P H + c_P \Phi^0)\sqrt{2} - \frac{i(s_0 H + c_P \Phi^0 + \sqrt{2}v')}{\sqrt{2}}\,, \\
h^+ &= c_+ G^+ - s_+ \Phi^+\,, \\
\phi^+ &= \frac{(s_+ G^+ + c_+ \Phi^+)}{i}\,, \\
\phi^{++} &= \frac{\Phi^{++}}{i}\,.
\end{aligned}$$

We use the following notation for the physical mass eigenstates: H and Φ^0 are neutral scalars, Φ_P is a neutral pseduscalars, Φ^+ and Φ^{++} are the charged and doubly charged scalars, and G^+ and G^0 are the Goldstone bosons that are eaten by the light W and Z bosons, giving them mass. Note that in defining the mass eigenstates we have factored out an i from ϕ. The mixing angles in the pseudoscalar and singly-charged sectors are easily extracted in terms of the vacuum expectation values:

$$\begin{aligned}
s_P &= \frac{2\sqrt{2}v'}{\sqrt{v^2+8v'^2}} \simeq \frac{2\sqrt{2}v'}{v}\,, \qquad & c_P &= \frac{v}{\sqrt{v^2+8v'^2}} \simeq 1 - \frac{4v'^2}{v^2} \\
s_+ &= \frac{2v'}{\sqrt{v^2+4v'^2}} \simeq \frac{2v'}{v}\,, \qquad & c_+ &= \frac{v}{\sqrt{v^2+4v'^2}} \simeq 1 - \frac{2v'^2}{v^2}
\end{aligned}$$

Diagonizing the mass terms for the neutral CP-even scalars gives the scalars mixing angle s_0, c_0 to leading order in v/f:

$$s_0 \simeq 2\sqrt{2}\frac{v'}{v}\,, \qquad c_0 \simeq 1 - 4\frac{v'^2}{v^2}\,.$$

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